# APPENDIX B

# STATISTICAL DISTRIBUTION USED IN RELIABILITY AND MAINTAINABILITY

## B-1. Introduction to statistical distribution

Many statistical distributions are used to model various reliability and maintainability parameters. The particular distribution used depends on the nature of the data being analyzed.

a. *Exponential and Weibull.* These two distributions are commonly used for reliability modeling – the exponential is used because of its simplicity and because it has been shown in many cases to fit electronic equipment failure data, and the Weibull because it consists of a family of different distributions that can be used to fit a wide variety of data and it models wearout (i.e., an increasing hazard function).

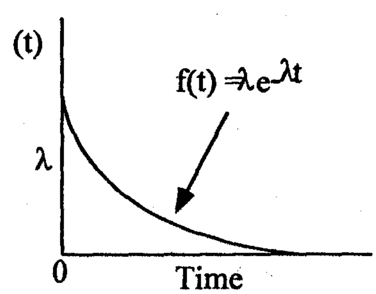
b. *Normal and lognormal.* Although also used to model reliability, the normal and lognormal distributions are more often used to model repair times. In this application, the normal is most applicable to simple maintenance tasks that consistently require a fixed amount of time to complete with little variation. The lognormal is applicable to maintenance tasks where the task time and frequency vary, which is often the case for complex systems and products.

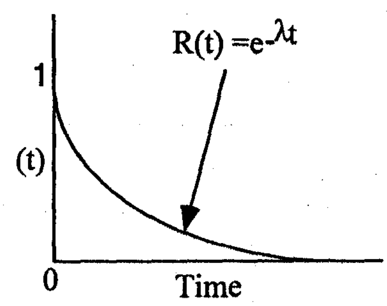
## B-2. The exponential distribution

The exponential distribution is widely used to model electronic reliability failures in the operating domain that tend to exhibit a constant failure rate. To fail exponentially means that the distribution of failure times fits the exponential distribution as shown in table B-1. The characteristics of the exponential distribution are listed in table B-2. figure B-1 shows the exponential pdf for varying values of λ.

*Table B-1. Summary of the exponential distribution*

| Probability Density Function | Reliability Function | Hazard Function |
|---|---|---|
| | | $h(t) = f(t)/R(t) = \lambda e^{-\lambda t}/e^{-\lambda t} = \lambda$ |



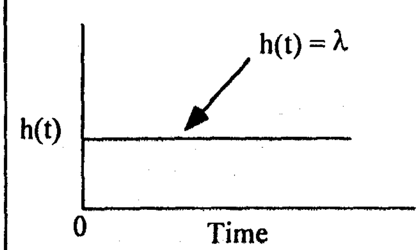



*Table B-2. Characteristics of the exponential distribution*

- It has a single parameter, λ, which is the mean. For reliability applications, λ called the failure rate.
- λ, the failure rate, is a constant. If an item has survived for t hours, the chance of it failing during the next hour is the same as if it had just been placed in service.
- The mean-time-between-failure (MTBF) = 1/ λ.
- The mean of the distribution occurs at about the 63rd percentile. Thus, if an item with a 1000-hour MTBF had to operate continuously for 1000 hours, the probability of success (survival) would be only 37%.

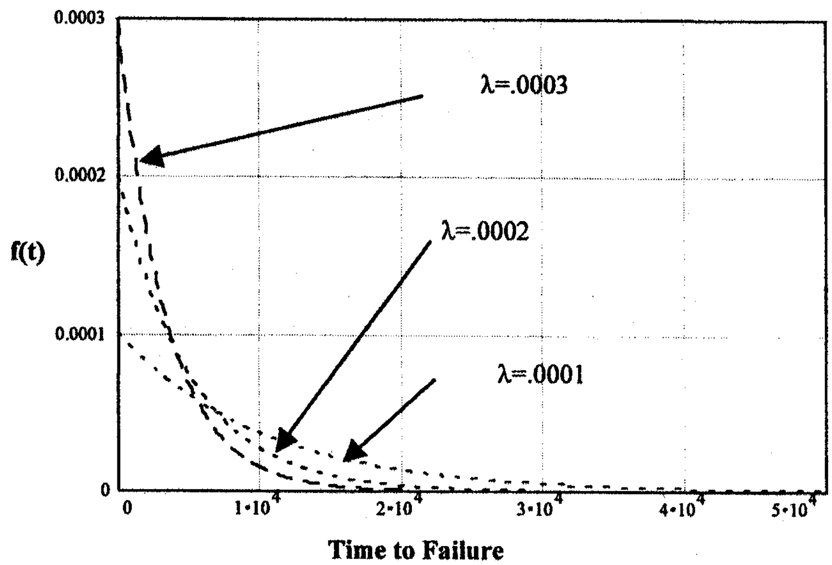



*Figure B-1. The exponential pdf for varying values of $\lambda$.*

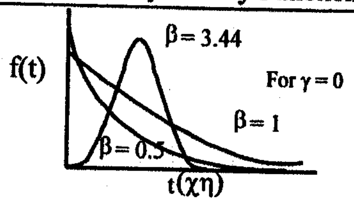
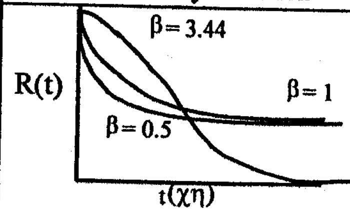
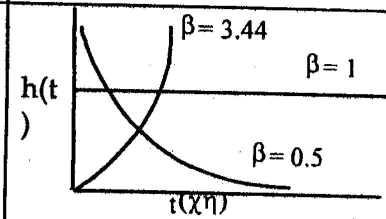
## B-3. The Weibull distribution

The Weibull distribution is an important distribution because it can be used to represent many different pdfs; therefore, it has many applications. The characteristics of the Weibull are shown in table B-3. The distribution is described in table B-4. Figure B-2 shows the 2-parameter Weibull pdf for different values of β and a given value of η.

*Table B-3. Characteristics of the Weibull distribution*

- It has 2 (β and η) or 3 (β, η, and γ) parameters.
  - The shape parameter, β, describes the shape of the pdf.
  - The scale parameter, η, is the 63rd percentile value of the distribution and is called the characteristic life. In some texts, θ is used as the symbol for the characteristic life.
  - The location parameter, γ, is the value that represents a failure-free or prior use period for the item. If there is no prior use or period where the probability of failure is zero, then γ = 0 and the Weibull distribution becomes 2-parameter distribution.
- β, η, and γ can be estimated using Weibull probability paper or software programs.
- When β = 1 and γ = 0, the Weibull is exactly equivalent to the exponential distribution.
- When β = 3.44, the Weibull closely approximates the normal distribution.

*Table B-4. Summary of the Weibull distribution*

| Probability Density Function | Reliability Function | Hazard Function |
|---|---|---|
|   |   |   |
| $(t) = \frac{\beta}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{\beta-1} \exp\left[-\left(\frac{t-\gamma}{\eta}\right)^{\beta}\right]$ | $R(t) = \exp\left[-\left(\frac{t-\gamma}{\eta}\right)^{\beta}\right]$ | $h(t) = \frac{\beta}{\eta}\left(\frac{t-\gamma}{\eta}\right)^{\beta-1}$ |

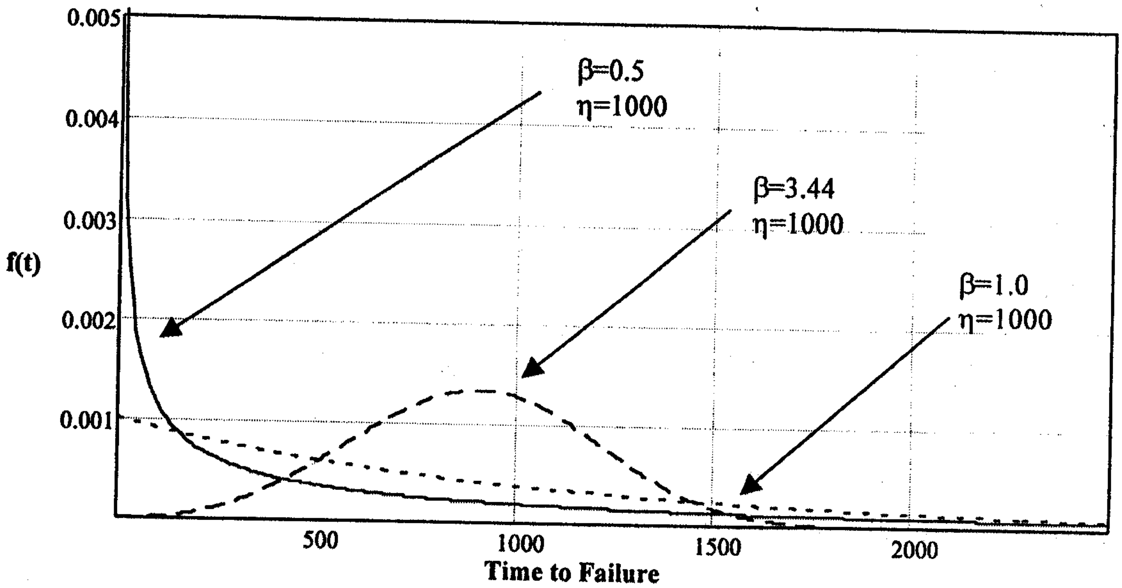



*Figure B-2. The two-parameter Weibull pdf for different values of β and a given value of η.*

## B-4. The normal distribution

The pdf of the Normal distribution is often called the bell curve because of its distinctive shape. The Normal distribution is described in table B-5. The characteristics of the Normal distribution are shown in table B-6. Figure B-3 shows the normal pdf for different values of σ and a fixed value of μ.

*Table B-5. Summary of the normal distribution*

| Probability Density Function | Reliability Function | Hazard Function |
|---|---|---|
| $f(t) = \frac{1}{\sigma\sqrt{2\pi}}\, e^{\frac{-(t-\mu)^2}{2\sigma^2}}$ | $R(t) = \int_t^{\infty} f(t)\,dt$ | $h(t) = \frac{f(t)}{R(t)}$ |

*Table B-6. Characteristics of the normal distribution*

| |
|---|
| • It has two parameters:<br>– The mean, $\mu$, is the 50th percentile of the distribution. The distribution is symmetrical around the mean.<br>– The standard deviation, $\sigma$, is a measure of the amount of spread in the distribution.<br>• If t has the pdf defined in figure B-5 and $\mu = 0$ and $\sigma = 1$, then t is said to have a standardized normal distribution.<br>• The integral of a distribution's pdf is its cumulative distribution function, used to derive the reliability function. The integral of the normal pdf cannot be evaluated using the Fundamental Theorem of Calculus because we cannot find a function for which the derivative equals $\exp(-x^2/2)$. However, numerical integration methods have been used to evaluate the integral and tabulate values for the standard normal distribution. |

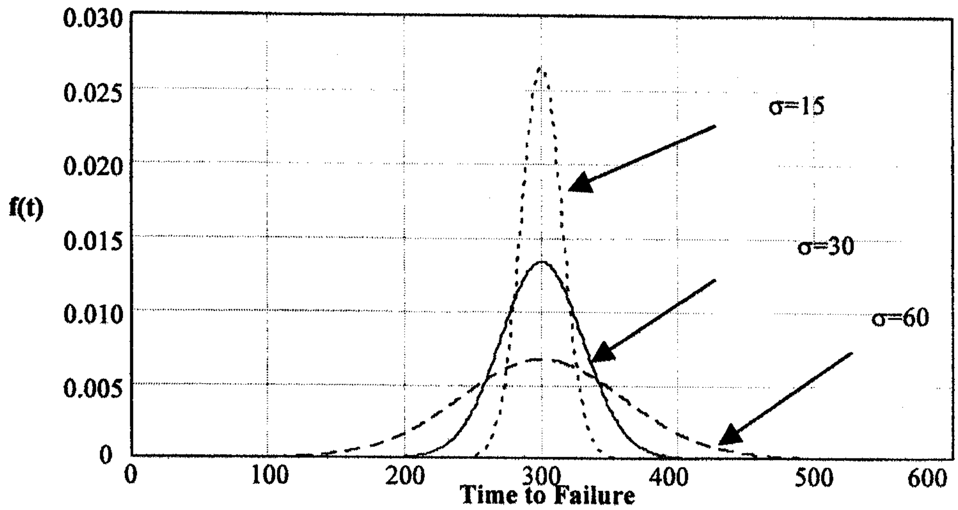



*Figure B-3. The normal pdf for varying values of $\sigma$ and a fixed $\mu$.*

## B-5. The lognormal distribution

The lognormal distribution is summarized in table B-7. The characteristics of the lognormal distribution are shown in table B-8. Figure B-4 shows the distribution for different values of $\mu$ and $\sigma$.

*Table B-7. Summary of the lognormal distribution*

| Probability Density Function | Reliability Function | Hazard Function |
|---|---|---|
| $f(t) = \frac{1}{\sigma t \sqrt{2\pi}} e^{\frac{-(\ln t - \mu)^2}{2\sigma^2}}$ | $R(t) = \int_t^{\infty} f(t)\, dt$ | $h(t) = \frac{f(t)}{R(t)}$ |

*Table B-8. Characteristics of the lognormal distribution*

| |
|---|
| • It has two parameters:<br>– The mean, $\mu$. Unlike the mean of the Normal distribution, the mean of the lognormal is not the 50th percentile of the distribution and the distribution is not symmetrical around the mean.<br>– The standard deviation, $\sigma$.<br>• The logarithms of the measurements of the parameter of interest (e.g., time to failure, time to repair) are normally distributed. |

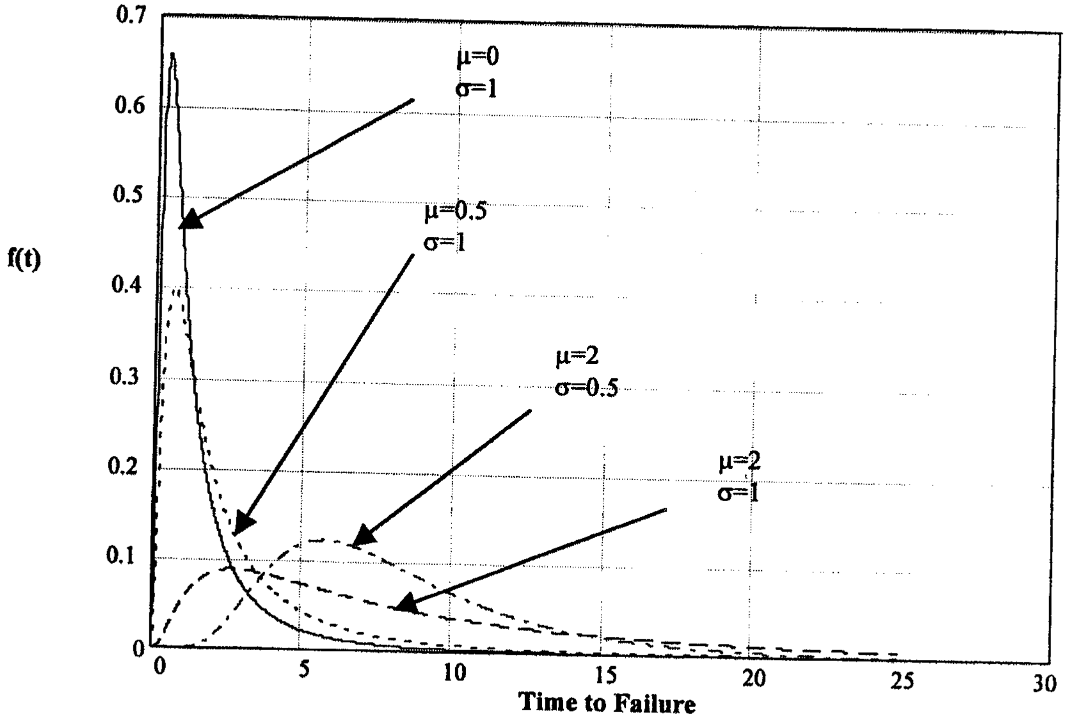



*Figure B-4. The lognormal pdf for different values of μ and σ.*